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Chiral symmetry and properties of hadron correlators in matter

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The constraints imposed by chiral symmetry on hadron correlation functions in nuclear medium are discussed. It is shown that these constraints implies the certain structure of the in-medium hadron correlators and lead to the cancellation of the order ρm_π term in the in-medium nucleon correlator. We also consider the effect of mixing of the chiral partners correlaton functions arising from the interaction of nuclear pions with corresponding interpolating currents. It reflects the phenomena of partial restoration of chiral symmetry. The different scenarios of such restorations are briefly discussed.

Keywords: chiral symmetry, mixing effect, hadron correlators, pions

PACS: 11.30.Rd, 11.55.Hx, 21.65.+f

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Chiral symmetry (CS) became nowadays one of the most important principles of the modern nuclear physics [1]. In the limit of massless quarks the QCD Lagrangian is symmetric with respect to the transformations belonging to the $SU(N) \times SU(N)$ chiral group. It is generally believed that this symmetry is spontaneously broken so that the ground state of the theory does not enjoy the symmetry of the underlying Lagrangian. In the case of $SU(2) \times SU(2)$ group, which is of relevance for nuclear physics, the Goldstone theorem implies the existence of 3 massless bosons which are responsible for the NN interactions. CS breaking manifests itself in the absence of chiral multiplets of the particles with the same masses but different parities, for example, $\rho - a_1$ or $\sigma - \pi$ mesons. In the language of the correlation functions the broken chiral symmetry means that the lowest pole positions of vector and axial vector correlators, describing the ρ and a_1 mesons , are different. Thus, the restoration of the symmetry in vacuum, which can happen in the academic limit when the number of the light flavors is more than 5 [2], results in the identity of the corresponding correlators which in turn leads to the same masses of the chiral partners. In matter the pattern of complete or partial restoration of CS is more complicated, as discussed below. The other manifestation of the spontaneously broken CS is the occurrence of the nonzero order parameters. One of the possible order parameters is the expectation value of the two quark condensate $\overline{q}q$. It is believed in the standard scenario of hadron interactions in vacuum that $q\overline{q}$=0 would imply the restoration of CS and therefore the masses of hadrons, larger part of which is due to CS breaking, should become much smaller compared to its observed values (we put aside more exotic possibilities of having spontaneously broken CS with significantly reduced or even vanishing two quark condensate [3]). One notes that the relationships between condensates, hadron masses and corresponding correlators are significantly more complicated in the presence of medium. For example, in the lukewarm pion gas the temperature dependence of the two quark condensate $\overline{q}q(T)$ and the in-medium nucleon mass is different [4] so that the change of the nucleon mass in the medium is not completely determined by the corresponding change of the quark condensate. Analogous conjecture is valid in nuclear matter [5]. We will discuss this latter case in more details

below. The other example is the issue of chiral symmetry restoration for the correlators of the chiral partners. In this case the restoration of the symmetry which means the identity of these correlators, in vacuum would imply the equality of the masses of chiral partners. The presence of matter makes things more complicated and the identity of the corresponding in-medium correlators does not nesseseraly means the degeneracy of the effective masses of chiral partners. The phenomena of axial-vector mixing established both at finite temperature [6] and and density [7] provides such an example. This mixing naturally leads to the several possible scenarios of CS restoration in matter. The identity of the masses of chiral partners at the point of restoration is only one of them. In this paper we address the issue of constraints which chiral symmetry imposes on the hadron correlators in nuclear medium. Some of the results presented in this paper were earlier discussed in short letters [7,8]. Here we present the formal aspects of calculations in greater details and give the extended discussion of the problems involved. The paper is organized as follows. In the next section we derive the expression for the leading chiral corrections for the in-medium hadron correlators. In the section 3 we consider the case of the nucleon correlators and discuss the relationships between density dependence of the two-quark condensate and nucleon mass. The next section is devoted to the phenomena of the mixing between the vector and axial-vector correlators and possible scenarios of CS restoration. In the section 5 we briefly discuss the leading chiral corrections for the two-quark condensate and in the next section we consider the effect of chiral mixing in the scalar-pseudoscalar channel. We summarize and conclude in the last section.

2. The dynamics of hadrons in nuclear medium is described by the corresponding correlators. Let's consider the case of the two-point correlators. It can be written in the form

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle \Psi | T\{J(x)J(0)\} | \Psi \rangle. \tag{1}$$

Where J is the interpolating current in the corresponding hadron channel and the matrix element is taken over the ground states of the system with finite density ρ. In each particular case the interpolating current should be supplied with the corresponding Lorenz and isospin

indices to specify the hadron channel considered. The position of the lowest pole and its behavior as the function of density determines the in-medium mass of hadron. The medium effects are included in the nuclear wave function Ψ. If this wave function describes the infinite system of non interacting nucleons than the above correlator reflects the dynamics of the probe hadron moving in some mean field formed by nuclear matter. To calculate the corrections to this rather crude picture one needs to take into account the nuclear pions which can be absorbed or emitted by nucleons and provide the interactions among them. It is convenient to use the Fock representation of the nuclear wave function.

$$\Psi = \Psi^A |A\rangle + \sum_a \Psi_a^A |A\pi_a\rangle + \sum_{a,b} \Psi_{a,b}^A |A\pi_a \pi_b\rangle + ... \tag{2}$$

Here A represents the state vector in the Fock space describing the nuclear system of A nucleons without pions, $|A\pi_a\rangle$ is the Fock state of the nuclear system with A nucleons plus one pion and $|A\pi_a \pi_b\rangle$ denotes the Fock state with A nucleons and two pions. The summation goes over the corresponding pion quantum numbers. We truncated the series taking into account the states with zero, one and two pions since the main contribution and leading chiral corrections are given by the matrix elements taken with these states. The states with larger number of pions would give the corrections of higher chiral dimension. Since the nuclear pions are virtual, all possible time orderings should be taken into account. Putting the above expression for the nuclear wave function into Eq.(1) and making use the identities

$$\langle \Psi^A | \Psi_{a,b}^A \rangle = \langle \Psi^A | a_a^+ a_b^+ | \Psi^A \rangle, \tag{3}$$

$$\langle \Psi_{a,b}^A | \Psi^A \rangle = \langle \Psi^A | a_a a_b | \Psi^A \rangle, \tag{4}$$

and

$$\langle \Psi_a^A | \Psi_b^A \rangle = \langle \Psi^A | a_a a_b^+ | \Psi^A \rangle \tag{5}$$

one can represent the in-medium hadron correlator by the sum of two pieces

$$\Pi(q) = \overset{\circ}{\Pi}(q) + \Pi^\pi(q) \tag{6}$$

Here the first term corresponds to the contributions from the system of noninteracting nucleons and can be written as follows

$$\overset{\circ}{\Pi}(q) = i \int d^4 x e^{ip \cdot x} \langle A | T\{J(x)J(0)\} | A \rangle, \tag{7}$$

whereas the second one describes the pionic corrections when the interpolating current interacts directly with the nuclear pions. Let's consider the specific part of this corrections where pion is first emitted and then absorbed by nuclear matter. The corresponding piece of the correlators can be represented in the form

$$\sum_{a,b} \int \frac{d^3 \mathbf{k}}{2\omega_k} \frac{d^3 \mathbf{k}'}{2\omega_k'} \langle \Psi^A | a^{a\dagger}(\mathbf{k}) a^b(\mathbf{k}') | \Psi^A \rangle \, i \int d^4 x e^{ip \cdot x} \langle A\, \pi^a(\mathbf{k}) | T\{J(x)J(0)\} | A\, \pi^b(\mathbf{k}') \rangle, \tag{8}$$

As we mentioned earlier due to virtual character of the nuclear pions all time orderings allowing pions to go backward or forward in time should be taken into account. These pieces of the correlator $\Pi^\pi(q)$ can be written as follows

$$\frac{1}{2} \sum_{a,b} \int \frac{d^3 \mathbf{k}}{2\omega_k} \frac{d^3 \mathbf{k}'}{2\omega_k'} \langle \Psi^A | a^a(\mathbf{k}) a^b(\mathbf{k}') | \Psi^A \rangle \, i \int d^4 x e^{ip \cdot x} \langle A | T\{V_\mu(x)\, V^\mu(0)\} | A\, \pi^a(\mathbf{k})\pi^b(\mathbf{k}') \rangle, \tag{9}$$

and

$$\frac{1}{2} \sum_{a,b} \int \frac{d^3 \mathbf{k}}{2\omega_k} \frac{d^3 \mathbf{k}'}{2\omega_{k'}} \langle \Psi^A | a^{a\dagger}(\mathbf{k}) a^{b\dagger}(\mathbf{k}') | \Psi^A \rangle \, i \int d^4 x e^{ip \cdot x} \langle A\, \pi^a(\mathbf{k})\pi^b(\mathbf{k}') | T\{V_\mu(x), V^\mu(0)\} | A \rangle \tag{10}$$

where $\omega_k = \sqrt{k^2 + m_\pi^2}$. One notes that the correlator $\overset{\circ}{\Pi}(p)$ with noninteracting nucleons was used for calculating of the in-medium properties of nucleon [9,10] and ρ-meson [11] in the framework of QCD sum rules. We consider the nuclear pions in the chiral limit. Since we are interested in the properties of hadron correlators which are related to chiral symmetry such as the lowest pole position in nuclear medium treating nuclear pions in the chiral limit seems to be a reasonable assumption in our case. Moreover, if one believes that the hadron mass generation mechanism is due to spontaneous breaking of CS then finite pion mass would act as a small perturbation. This conjecture is partly supported by the results of Ref. [12] where it was shown that in the chiral limit nuclear properties (at least the static ones) experience little changes compared to the real case. The use of soft-pion theorem gives

$$i\langle A\,\pi^a(\mathbf{k})|T\{J(x),J(0)\}|A\,\pi^b(\mathbf{k}')\rangle \simeq \frac{-i}{f_\pi^2}\langle A|\left[Q_5^a,\left[Q_5^b,T\{J(x),J(0)\}\right]\right]|A\rangle, \qquad (11)$$

The other matrix elements can also be reduced to similar expressions in the same manner. Now the part of the correlator describing the pionic corrections can be written as follows

$$\Pi^\pi = \frac{-i}{2f_\pi^2}\sum_{a,b}\int \frac{d^3\mathbf{k}}{2\omega_k}\frac{d^3\mathbf{k}'}{2\omega_{k'}}\langle\Psi^A|((a_a^+(\mathbf{k})a_b^+(\mathbf{k}') + a_a^+(\mathbf{k})a_b^+(\mathbf{k}') +$$

$$2a_a^+(\mathbf{k})a_b(\mathbf{k}'))|\Psi^A\rangle\, i\int d^4x\, e^{ip\cdot x}\langle A|[Q_5^a,[Q_5^a,T\{J(x),J(0)\}]]|A\rangle, \qquad (12)$$

The first integral in this expression is proportional to the result of expansion of the matrix element $\langle\Psi^A|\frac{1}{2}\pi^2(0)|\Psi^A\rangle$ in terms of creation and annihilation operators. Here $\pi(0)$ is the pion field operator and the normal ordering is assumed. To the first order in density this matrix element is

$$(2\pi)^3\langle\Psi^A|\frac{1}{2}\pi^2(0)|\Psi^A\rangle m_\pi^2 \simeq (2\pi)^3 m_\pi^2\langle N|\pi^2(0)|N\rangle\rho \simeq \rho\overline{\sigma}_{\pi N} \qquad (13)$$

In this expression the matrix element is taken over the nucleon states and $\overline{\sigma}_{\pi N}$ is the leading nonanalytic part of the pion-nucleon sigma term $\sigma_{\pi N}$. The appearance of this nonanalytic term can qualitatively be understood as follows. The chiral expansion of $\sigma_{\pi N}$ reads as

$$\sigma_{\pi N} = Am_\pi^2 - \frac{9}{16}(\frac{g_{\pi N}}{2M_N})^2 m_\pi^3 + ... \qquad (14)$$

First, analytic term is proportional to some constant which contains a counterterm needed to regularize the short range contributions. In contrast, the nonanalytic term is due to long distance contribution of the pion cloud. Being long-ranged this term is governed by CS and thus should enter the expression describing the leading chiral corrections, where the main role is played by the nuclear pion cloud. The actual type of the chiral expansion of the in-medium hadron correlator depends on the commutation relation between chiral generator Q_5^a and hadron interpolating current chosen. In the next section we consider the case where the interpolating current relates vacuum and the states with the nucleon quantum numbers.

    3. The renormalization of nucleon mass in dense matter is one of the main issues in nuclear physics for a long time. Many recent studies of the in-medium nucleon dynamics have

to large extent been focused on the relations between the phenomena of partial restoration of CS and changes of nucleon properties in matter. As we already mentioned this hidden symmetry is characterized by some order parameter which in our case is provided by the nonzero vacuum expectation value of the two quark condensate $\overline{q}q$. Matter can influence the properties of the QCD vacuum and thus change the two quark condensate. It is usually believed that, in general, the reduction of $< \overline{q}q >$ is related in some way to the effect of partial restoration of chiral symmetry. To the first order in the density the evolution of the two-quark condensate is given by [9,10]

$$\langle \Psi | \overline{q}q | \Psi \rangle = \langle 0 | \overline{q}q | 0 \rangle (1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho), \tag{15}$$

where $|\Psi\rangle$ denotes our nuclear matter ground state. Much attention has recently been paid to the issue of the higher density corrections to this result [13,14]. At present there is a qualitative agreement that the contributions of the higher order terms are moderate up to the normal nuclear matter density. On the contrary, there has been much less discussions on how the changes of the two quark condensate affect the in-medium particle properties and whether any change of $\overline{q}q$ is in the one-to-one correspondence with the CS restoration phenomena. This problem was first discussed in ref. [5]. The reasoning was the following. One considers the scalar self-energy of a nucleon in nuclear matter. Let this self-energy depend in some way on the quark condensate. Then one should be a piece in the the self-energy proportional to the second term in the above written expression for the in-medium quark condensate. At moderate densities the nucleon self-energy can be represented by the product of the scalar part of the NN amplitude and nuclear density ρ so there will be a piece of the NN amplitude which is proportional to $\frac{\sigma_{\pi N}}{m_\pi^2}$. Making use of the chiral expansion of the pion-nucleon sigma term one can easily see the the NN amplitude contains the term of order m_π not allowed by chiral counting rules [15] according to which there is no term of this order in the chiral expansion of the NN amplitude. Thus one can conclude that there is no direct correspondence between changes of the quark condensate in nuclear matter and the in-medium nucleon properties. One notes that this conclusion is in complete accord with the

results obtained by Cohen et al. [16] where it was pointed out that, in order to be consistent with chiral symmetry, the expression for the nucleon mass in vacuum should contain no terms of order m_q times by the logarithm of the quark mass whereas these terms present in quark condensate. It was demonstrated in [16] how, using the QCD sum rules, one can remove the unwanted pieces. Technically such terms in the Operator Product Expansion (OPE) can be canceled by the corresponding piece of the phenomenological side of QCD sum rules provided a careful treatment of the continuum, including the low-momentum πN states, is made. As we show below, similar cancellation takes place in nuclear matter. It is again convenient to use the QCD sum rules approach to demonstrate how this cancellation appears in the case of nuclear matter. In QCD sum rules one relates the characteristics of QCD vacuum and the phenomenological in-medium nucleon spectral density. The OPE consists of certain set of local operators responsible mainly for short range effects. The effects of the low-momentum pions are long-ranged and should, therefore, stem from the phenomenological representation of the in-medium nucleon correlator. The correlator of two nucleon interpolating currents can be written as follows

$$\Pi(p) = i \int d^4 x e^{ip \cdot x} \langle \Psi | T\{\eta(x) \overline{\eta}(0)\} | \Psi \rangle. \tag{16}$$

Assuming the Ioffe choice [17] of the nucleon interpolating current, making use of the transformation property of this current

$$[Q_5^a, \eta(x)] = -\gamma_5 \frac{\tau^a}{2} \eta(x). \tag{17}$$

one can get the following chiral expansion of the in-medium nucleon correlator

$$\Pi(p) \simeq \overset{\circ}{\Pi}(p) - \frac{\xi}{2} \left(\overset{\circ}{\Pi}(p) + \gamma_5 \overset{\circ}{\Pi}(p) \gamma_5 \right), \tag{18}$$

where we defined

$$\xi = \frac{\rho \overline{\sigma}_{\pi N}}{f_\pi^2 m_\pi^2} \tag{19}$$

and $\overset{\circ}{\Pi}(p)$ is the nucleon correlator in the chiral limit. It is useful to to decompose the correlator into three terms with the different Dirac structures [10]

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$$\Pi(p) = \Pi^{(s)}(p) + \Pi^{(p)}(p)\not{p} + \Pi^{(u)}(p)\not{u}, \tag{20}$$

where u^μ is a unit four-vector defining the rest-frame of nuclear system. One can see that only the piece $\Pi^{(s)}(p)$ gets affected by the chiral corrections of order ρm_π. Given the fact that the OPE for $\Pi^{(s)}(p)$ involves terms transforming like two-quark condensate $\overline{q}q$ whereas $\Pi^{u(p)}(p)$ contains chirally invariant matrix elements such as $G_{\mu\nu}G^{\mu\nu}$ or q^+q this result looks quite natural. Splitting, as it is usually done in the QCD sum rules method, the phenomenological expression of the nucleon correlator into pole and continuum parts one can obtain

$$\Pi(p) \simeq \Pi_{pole}(p) - \frac{\xi}{2}\gamma_5 \Pi_{pole}(p)\gamma_5 + \left(1 - \frac{\xi}{2}\right)\overset{\circ}{\Pi}_{cont}(p) - \frac{\xi}{2}\gamma_5 \overset{\circ}{\Pi}_{cont}(p)\gamma_5, \tag{21}$$

Where we denoted $\Pi_{pole}(p) \simeq (1 - \frac{\xi}{2})\overset{\circ}{\Pi}_{pole}(p)$ The explicit expression of the pole term has the following form [10]

$$\Pi_{pole}(p) = -\lambda^{*2}\frac{\not{p} + M^* + V\gamma_0}{2E(\mathbf{p})[p^0 - E(\mathbf{p})]}, \tag{22}$$

Here M^* is the in-medium nucleon mass including the scalar part of the self energy and λ^* is the coupling of the nucleon interpolating current with the corresponding lowest state. According to the standard ideology of QCD sum rules the OPE and phenomenological sides should be matched with some weighting functions. Then one writes the three independent sum rules, one for each Dirac structure

$$-\left(1 - \frac{\xi}{2}\right)\lambda^{*2}M^*\int d^4p \frac{w(p)}{2E(\mathbf{p})[p^0 - E(\mathbf{p})]} \simeq (1 - \xi)\int d^4p\, w(p)\left[\overset{\circ}{\Pi}^{(s)}_{OPE}(p) - \overset{\circ}{\Pi}^{(s)}_{cont}(p)\right],$$

$$-\left(1 + \frac{\xi}{2}\right)\lambda^{*2}\int d^4p \frac{w(p)}{2E(\mathbf{p})[p^0 - E(\mathbf{p})]} \simeq \int d^4p\, w(p)\left[\overset{\circ}{\Pi}^{(p)}_{OPE}(p) - \overset{\circ}{\Pi}^{(p)}_{cont}(p)\right], \tag{23}$$

$$-\left(1 + \frac{\xi}{2}\right)\lambda^{*2}V\int d^4p \frac{w(p)}{2E(\mathbf{p})[p^0 - E(\mathbf{p})]} \simeq \int d^4p\, w(p)\left[\overset{\circ}{\Pi}^{(u)}_{OPE}(p) - \overset{\circ}{\Pi}^{(u)}_{cont}(p)\right]. \tag{24}$$

Taking the ratio of these sum rules one can get the needed cancellation in the effective mass and vector self energy and bring the in-medium nucleon QCD sum rules in an agreement with the chiral symmetry constraints. The numerical value of $\overline{\sigma}_{\pi N}$ is about -20MeV to be compared with the numerical value of the entire pion-nucleon sigma-term $\sigma_{\pi N} = 45$MeV, so that the numerical effect is quite significant. One notes that from the chiral expansion of the

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nucleon correlator one can easily see that the factorization approximation, sometimes used for the four quark condensate, cannot be valid in matter since in does not satisfy the chiral symmetry requirements. This condensate enters the OPE of $\Pi^{p(u)}(p)$ which, being chiral invariant, is not affected by chiral corrections. When the factorization is used the chirally noninvariant product of two two-quark condensates appears in the OPE of $\Pi^{p(u)}(p)$ bringing in the terms of order ρm_π which are inconsistent with chiral symmetry.

4. In this section we consider the in-medium correlation function of the vector currents. First, consider the case of the isovector-vector currents. The general expression for this correlator can be written as follows

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ip\cdot x} \langle \Psi | T\{J_\mu(x) J_\nu(0)\} | \Psi \rangle. \qquad (25)$$

Where J_μ is the isovector vector interpolating current. The lowest pole of this correlator corresponds to the ρ-meson contribution. The problem of how medium with nonzero density and/or temperature alters the ρ-meson properties compared to those in vacuum attracts much attention nowadays [18]. Due to relatively large width it decays basically inside nuclear interior so the spectrum of the produced dileptons can carry, at least in principle, the direct information about the modifications of the ρ-meson mass and width in matter. Such modification may be related with partial restoration of CS. The scaling relations proposed in [19] suggested that the ρ-meson mass should decrease in matter following the behavior of the chiral order parameter. First calculations, based on QCD sum rules [11] indeed supported this conjecture and predicted the drop of the ρ-meson mass by 20% at the normal nuclear density. One notes that in Ref. [11] the nuclear medium was represented as a system of the noninteracting nucleons and for the phenomenological spectral density the pole-plus-continuum anzats was used. Besides, 4-quark condensate was approximated by the corresponding factorized expression, which, as one could see in the previous section, violates the chiral symmetry constraints. In more recent calculations [20] the phenomenological spectral density was treated in more realistic way taking into account ρN scattering process. These calculations demonstrated that more realistic model of the spectral density leads to

much less pronounced alternations of the ρ-meson mass compared to the pole-plus-continuum model. Moreover, some phenomenological models predict even the increase of ρ-meson mass in matter [21]. One can thus conclude that this issue is not completely settled. The other, at least theoretical, way to look at the phenomena of CS restoration is to study the correlators describing the in-medium dynamics of the chiral partners. As we have already mentioned the correlators of the chiral partners, in our case the correlators of the vector and axial-vector currents, should become identical in the chirally restored phase. Thus one can expect that these correlators get mixed when the symmetry is only partially restored. In other words the difference between the correlators of the chiral partners, being in some since an order parameter, tends to become smaller with increasing density and/or temperature. The effect of chiral mixing indeed takes place both at finite temperatures [6] and densities [7]. We note that in the case of nuclear density with zero temperature the "axial phase" of the vector correlator should be accompanied by particle-hole excitation of the nucleus to preserve the total isospin. This point reflects the difference between the cases of finite temperature and finite density although it should rather be viewed as the "pure nuclear" effect. From the CS point of view the chiral mixing is the manifestation of the same physics both at finite T and ρ. We note that in the real case of the heavy ion collisions the chiral mixing may happen without such particle-hole intermediate excitation. The axial vector correlator contains the contribution from pion and axial-vector meson A_1 which is a chiral partner of the ρ-meson. Being a Goldstone boson, pion experiences little changes in matter up to the point of chiral phase transition. Pion decay constant F_π scales with density like the order parameter and can be fixed relatively well up to the density close to normal nuclear one. It is much more difficult to describe the in-medium properties of the A_1 meson. This meson can decay into three pions, each of them can interact with nuclear medium and with each other. All that makes it very difficult to estimate the correlator of the axial-vector current with sufficient accuracy. This in turn significantly increases the size of uncertainties related to the calculation of the in-medium mass of the ρ-meson since, due to mixing property, the vector correlator in nuclear medium acquires the additional singularity describing the contribution

of the A_1 meson which is multiplied by the corresponding, in general unknown, residue. One notes that we discuss the idealized case of the ρ meson at rest. In reality they move and it may result in significant effect [22,23] somehow relaxing the constraints following from CS. Making use of the standard commutation relation of current algebra $\left[Q_5^a, J_\nu^b\right] = i\epsilon^{abc} A_\nu^c$ and putting it in the general expression for the correlator of the vector currents Π_V one can get

$$\Pi_V = \overset{\circ}{\Pi}_V + \xi(\overset{\circ}{\Pi}_V - \overset{\circ}{\Pi}_A) \tag{26}$$

and similar expression for the axial vector correlator Π_A

$$\Pi_A = \overset{\circ}{\Pi}_A + \xi(\overset{\circ}{\Pi}_A - \overset{\circ}{\Pi}_V) \tag{27}$$

Here we defined

$$\xi = \frac{4\rho\overline{\sigma}_{\pi N}}{3f_\pi^2 m_\pi^2}. \tag{28}$$

and denoted $\overset{\circ}{\Pi}_V (\overset{\circ}{\Pi}_A)$ the correlator of the vector (axial) currents calculated in the approximation of the noninteracting nucleons with finite density. As one can see from the above equations the correlators get mixed when soft pion contributions are taken into account. We note that this statement is model independent and follows solely from CS. In contrast, most of the studies of the ρ meson in medium done so far focused on the calculations of different terms of the ρ meson self energy and had to explore some model dependent assumptions. On the other hand much less attention has been paid on the general chiral structure of the corresponding correlators. It is worth emphasizing that CS alone cannot predict the actual behavior of the in-medium ρ meson mass. Instead, chiral symmetry implies that the correlators of the chiral partners acquire, due to mixing, the additional singularities. It is an additional pole when the soft pion approximation is used, but in the general case of pions with nonzero 3-momentum the axial correlator exhibits a cut. These singularities may manifest themselves in the, for example, dilepton spectrum, produced in the heavy-ion collisions. It means that the spectrum may show the additional enhancement at the energy region close to the mass of the A_1 meson, besides that at the mass of ρ meson. It is interesting to note

that the complete mixing occurs at densities $\rho \simeq 2.5\rho_0$. It qualitatively agrees with the results of the actual calculations of the chiral phase transition but the soft pion approximation is certainly too crude to be used at high densities. The phenomena of mixing suggests few possible ways of how chiral symmetry restoration occurs. One notes that these scenarios are similar to those found earlier for the case of finite temperatures [24]. Firstly, the in-medium masses or, equivalently, the lowest singularities of the corresponding correlators may become the same at the point of restoration. Secondly, both correlators may exhibit two poles of the same strength corresponding to the ρ and A_1 mesons. In this scenario the identity of the correlators does not lead to the same masses of the chiral partners at the point of restoration. In these two scenarios it is implicitly assumed that the mesons still retain its individuality even at high densities. In other words, the in-medium widths of the the ρ and A_1 mesons are small compared to the mass difference of the mesons. One could suggest the other scenario where the widths are of the order of ρ - A_1 mass difference so that the spectral functions get smeared over the energy region, covering both mesons and thus it no longer makes sense to discuss the individual quantum states with the mesonic quantum number when the density is high enough. Due to relatively firmly established tendency of the ρ and A_1 meson widths to grow with density this third scenario seems most realistic although neither of the three scenarios can be ruled out. One needs to mention that the issue of the experimental study of the CS restoration using the chiral mixing relations is completely open question, since it is very difficult to measure the axial vector correlator.

One notes that the ω meson being an isospin singlet is not affected by the chiral corrections. It follows from the fact that the commutator of the isoscalar vector current with the axial charge Q_a^5 is zero. That in turn means that the case of ω meson is in some sense cleaner than the one with ρ meson since there is no chiral mixing. Of course, ω meson in medium is affected by many other nuclear effects but at least this source of uncertainties is absent. Now a few remarks concerning the general case of matter with nonzero density and temperature are in order . This regime is the one which is realized in the genuine heavy ion collisions. Since from the CS point of view the system with finite temperature does not differ from from

the one with finite density it is natural to expect that the phenomena of chiral mixing exists in the medium with finite temperature and density with separate contributions from the virtual nuclear and thermal pions. One could also expect some sort of mixed contribution when pions, being emitted by thermal bath, are then absorbed by nuclear medium. The analytical structure of the corresponding correlators becomes more complicated and may contain singularities due to both thermal and nuclear pions. These and related questions will be the subject of the forthcoming publication.

5. In this section we briefly address the issue of the chiral corrections to the lowest order result for the in-medium two-quark condensate. Let's first illustrate the derivation of the lowest order result. Assuming that the condensate at finite density can be represented as the sum of the vacuum contribution and the contribution coming from the system of the noninteracting nucleons with constant density one can write

$$\langle A|\overline{q}q|A\rangle_\rho = \int d^3x \langle A|\overline{q}q(x)|A\rangle = \langle 0|\overline{q}q|0\rangle + \rho \int d^3x \langle N|\overline{q}q(x)|N\rangle$$
$$= \langle 0|\overline{q}q|0\rangle(1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2}\rho), \tag{29}$$

Here we have used the definition of the πN sigma term and GOM relation. The corrections to this lowest order expression which are due to mesons of different type responsible for nuclear interaction, were studied in the number of papers [13,14]. One notes that, while the lowest order expression is model independent, the higher order contributions inevitably call for model dependent assumptions. In this section we are specifically interested in corrections which solely come from CS and, in some sense, are model independent . They can be obtained if we replace the T-product of two interpolating fields in Eq. (25) by the operator $\overline{q}q$. Following outlined above procedure and calculating corresponding commutators one can get

$$\langle A|\overline{q}q|A\rangle_\rho = \langle 0|\overline{q}q|0\rangle(1 - \frac{\rho\sigma_{\pi N}}{f_\pi^2 m_\pi^2}(1 - \frac{\rho\overline{\sigma_{\pi N}}}{f_\pi^2 m_\pi^2})), \tag{30}$$

One can see from the above expression that pion corrections result in a decrease the overall correction to the vacuum value of the two quark condensate by 12-15% depending on the

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value accepted for $\overline{\sigma_{\pi N}}$. In the other terms, the leading chiral corrections to the two quark condensate are entirely governed by the long-ranged part on the nuclear pion field. As was earlier mentioned the soft pion contribution to the in-medium two-quark condensate is not related to the chiral symmetry restoration but it is always useful to estimate the contribution of such terms since in gives the qualitative idea about the relative size of the corrections which are not due to soft pions and could thus can be relevant when the symmetry restoration related problem is considered. Now the remark concerning the issue of the two quark condensate in finite nuclei is in order. In finite nuclei density is no longer a constant but a function of the nuclear radius. Thus the two quark condensate effectively becomes nonlocal. Besides, this "nuclear" nonlocality , there is also "vacuum" nonlocality stemming from the fact that the standard two quark condensate is the first term of the short-range expansion of the nonlocal expression $\overline{q}q(x)$. These two nonlocalities should , in principle, be treated on the same ground. One notes, that in the processes where time plays important role as in heavy ion collisions, the time dependence of the quark condensate should also be included [25]. Thus we see that in the real situation the in-medium quark condensate becomes model dependent even in the lowest order in nuclear density.

6. In this section we briefly consider the mixing of the other type of chiral partners, namely the σ-π pair. The corresponding relations can be obtained straightforwardly from the general expression of the current-current correlator. Making use of the current algebra commutation relation

$$[Q_5^a, \pi^b] = -\delta^{ab} i\sigma. \tag{31}$$

and

$$[Q_5^a, \sigma] = -i\pi^a. \tag{32}$$

one can get the mixing relations

$$\Pi_\pi = (1 + \xi)\overset{\circ}{\Pi}_\pi - \xi(\overset{\circ}{\Pi}_\sigma) \tag{33}$$

and

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$$\Pi_\sigma = (1 + \xi)\overset{\circ}{\Pi}_\sigma - \xi(\overset{\circ}{\Pi}_\pi) \tag{34}$$

Here the mixing parameter is

$$\xi = \frac{2\rho\overline{\sigma}_{\pi N}}{3f_\pi^2 m_\pi^2}. \tag{35}$$

One can see that the effect of σ-π mixing is less pronounced at the normal nuclear density than in the case of ρ-A_1 mixing. The point of the complete restoration of chiral symmetry should, of course, be the same for all kinds of the chiral partners but the "velocity" of approaching to this point may well be different. We note that via the mixing with σ the pion polarization operator acquires the additional width. Similar to the case of the ρ-A_1 system the mixing in the pseudoscalar-scalar channel practically means that the correlators exhibit the singularities which are dictated by CS and should be taken into account regardless of the model used to describe the concrete hadronic processes. However, the effect of the σ-π mixing can probably be observed at relatively large densities. The case worth studying is deeply bound pion states [26,27] in heavy nuclei recently observed in $(d,^3He)$ reaction [28]. The other possible method to observe the effect of mixing and closely related phenomena of CS restoration is to measure the process of two pion production in hadron-nucleus interactions. Such kind of experiment has recently been made [29] on number of nuclei ranging from deuteron and up to heavy targets. The results of the measurements indicate the strong enhancement of the $\pi^+\pi^-$ pairs yield (in I=J=0 state) with the increase of the nuclear mass number. One notes that the results of recent theoretical paper [30] support the idea that the large enhancement of the $\pi^+\pi^-$ production near threshold is due to partial restoration of CS. The σ-π mixing , considered in the present paper is very likely responsible for the significant part of this enhancement.

Conclusion

We considered the relationships between chiral symmetry and properties of the in-medium hadron correlators. It was demonstrated that chiral symmetry imposes the important constraints on the correlators. In the case of the nucleon correlator it does not allow the terms

linear in pion mass to appear in the expression for the nucleon mass. It also fixes the certain properties of the pieces of the nucleon correlator with the different Dirac structures. The other important consequence of the chiral symmetry is that the factorization approximation, sometimes used to parameterize the in medium four quark condensate is not valid. In the case of the mesonic chiral chiral partners such as σ-π and ρ-A_1 pairs chiral symmetry results in the mixing of the corresponding correlators similar to those established earlier for the case of finite temperatures. This gives rise to the additional singularities of the correlators and may lead to the enhancement of experimental spectra in threshold region. This mixing can be viewed as the indication toward the restoration of chiral symmetry. It also suggests several possible scenarios of such a restoration.

ACKNOWLEDGMENTS

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